Section 3.2 - Differentiability

A function will not have a derivative at a point P(a, f(a)) where the slopes of the secant lines,

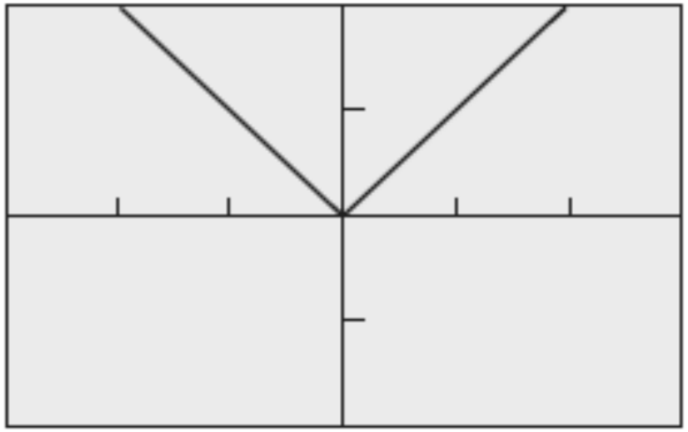
$$\frac{f(x) - f(a)}{x - a},$$

fail to approach a limit as x approaches a.

4 situations...

Example

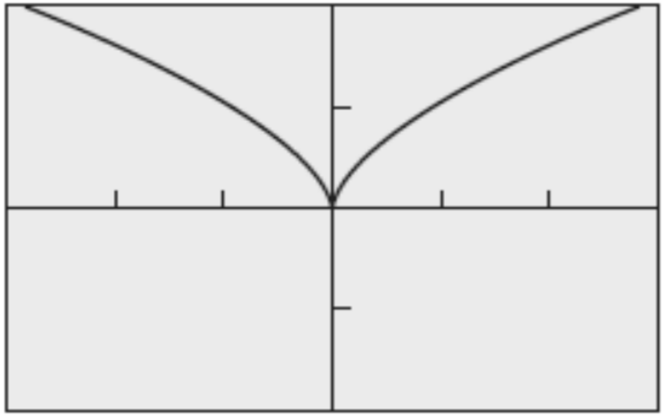
corner



[–3, 3] by [–2, 2]

1.) A corner, where the one-sided derivatives differ : Example, $f(x) = |x|$

2. Cusp - where the slopes of the secant lines approach ∞ from one side and $-\infty$ from the other

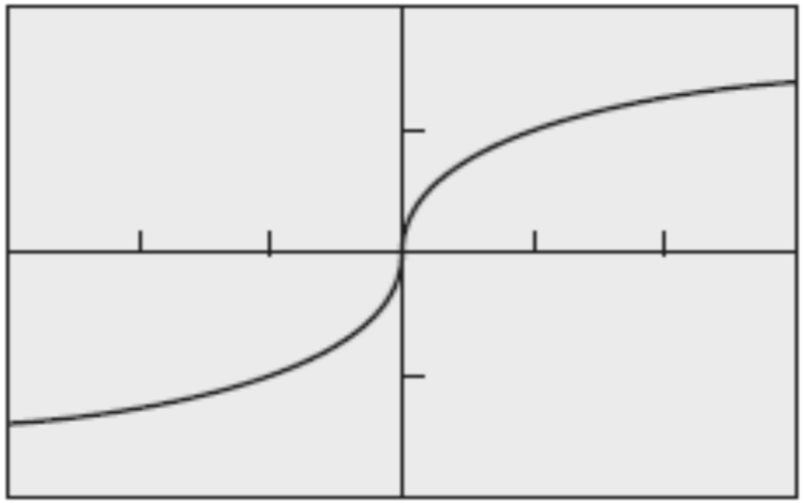


[–3, 3] by [–2, 2]

example $y = x^{2/3}$

3. A Vertical Tangent - where the slopes of the secant lines approach either ∞ or -∞ from both sides

There is a vertical tangent line at $x = 0$

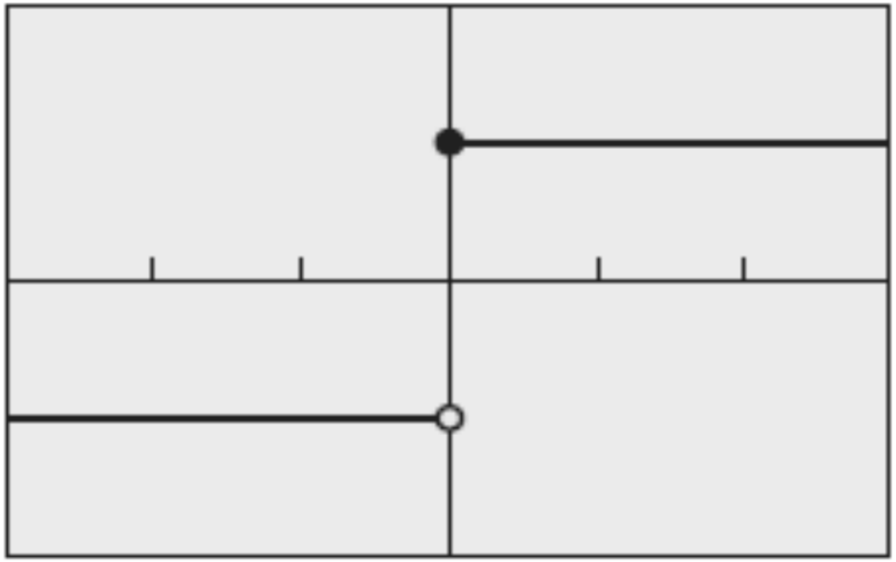


[–3, 3] by [–2, 2]

example $y = \sqrt[3]{x}$

4.) A discontinuity - which will cause one or both of the one-sided derivatives to be non-existent.

There is a discontinuity at x = 0



[–3, 3] by [–2, 2]

Example

Find all points in the domain of $f(x) = |x - 2| + 3$ where f is not differentiable.

Differentiability implies local linearity

- when you zoom in really really close to a point of tangency, the function looks more and more like the tangent line at that point!

  - differentiable curves will straighten out when we zoom in on them at a point of differentiability

If a function is differentiable at a point, it is locally linear!!!

Slope from left MUST equal the slope from the right

Tangent lines exist at that point as well

$y = |x|$ at $x = 0$

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Explain why there is no derivative at $x = 0$?

Explain why $y = x^{2/3}$ is not differentiable at $x = 0$.